EX-B

MECHANICS OF MATERIALS

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6.3 FLEXURAL STRESS IN LINEARLY ELASTIC BEAMS

In the previous section, assumptions were made about the geometry of deformation of slender beams, and an expression for the resulting extensional strain ϵ_x was derived, Eq. 6.3. The corresponding normal stress in beams, σ_x, is often called the *flexural stress*. To obtain an expression for the flexural stress, we need to consider the material behavior, that is, the stress-strain-temperature behavior of the material. To simplify our initial study of stresses in beams, let us assume that the material is linearly elastic and isotropic, and that the temperature remains constant. Then, the following two assumptions permit us to determine the flexural stress σ_x:

1. The material obeys Hooke's law, Eq. 2.32a, with $\Delta T = 0$.
2. The transverse normal stresses, σ_y and σ_z, may be neglected in comparison with the primary normal stress, σ_x.

By combining these two assumptions, we find that the uniaxial stress-strain equation

$$\sigma_x = E\epsilon_x \tag{6.6}$$

applies to bending of linearly elastic beams. When Eqs. 6.3 and 6.6 are combined, we obtain the following expression

$$\sigma_x = \frac{-Ey}{\rho} \tag{6.7}$$

If E = const, or if $E = E(x)$, the normal stress on a cross section is linear in y, as given by Eq. 6.8 and indicated in Fig. 6.10.[3]

$$\boxed{\sigma_x(x,y) = \frac{-E(x)\,y}{\rho(x)}} \tag{6.8}$$

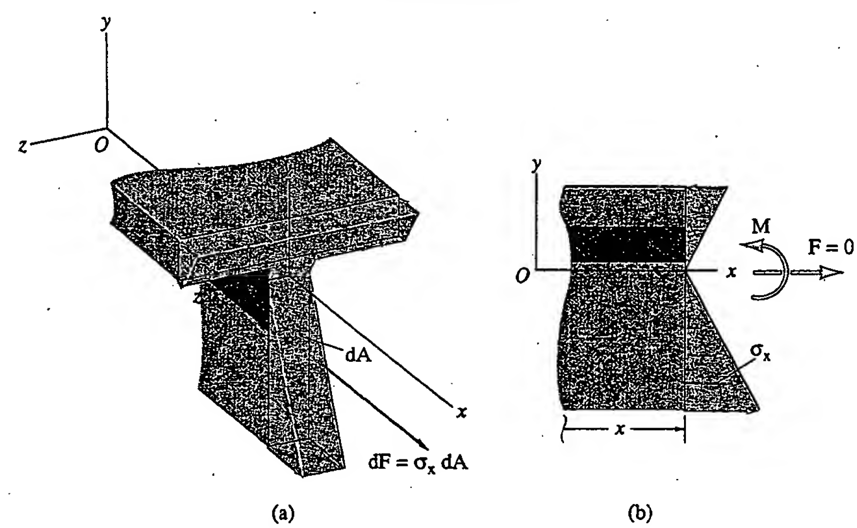


FIGURE 6.10 The flexural stress distribution at a cross section where $\rho(x)$ is positive.

[3]In Section 6.5 we will consider stresses in nonhomogeneous beams, that is, stresses in beams that are made of more than one material.

As indicated in Fig. 6.10, the stress resultants that are related to the normal stress σ_x acting on the cross section are:

$$F(x) = \int_A \sigma_x \, dA, \qquad M(x) = -\int_A y\sigma_x \, dA \tag{6.9}$$

A positive moment produces compression in the $+y$ fibers of the beam.

In Section 9.4 we will consider axial deformation combined with bending, but, for the present discussion of bending alone, let $F \equiv 0$. Therefore, substituting Eq. 6.8 into Eqs. 6.9, we get

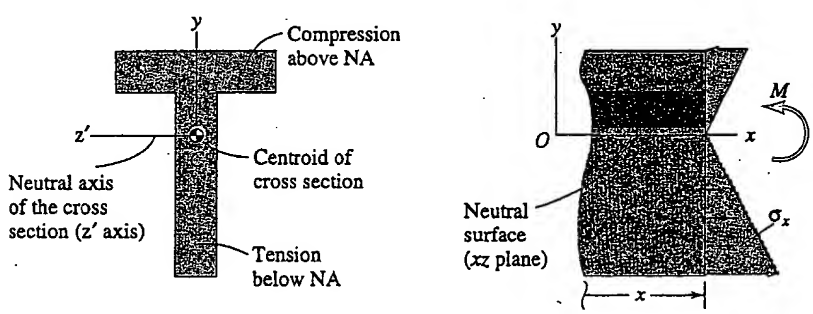
$$F = -\frac{E}{\rho}\int_A y \, dA = 0, \qquad M = \frac{E}{\rho}\int_A y^2 \, dA \tag{6.10}$$

The integrals appearing in Eqs. 6.10 are section properties that are defined in Appendix C:

$$\int_A dA = A, \qquad \int_A y \, dA = \bar{y}A, \qquad \int_A y^2 \, dA = I_z \tag{6.11}$$

where A is the cross-sectional area, $\bar{y}$ is the y coordinate of the *centroid* of the cross section, and I_z is the *area moment of inertia* about the z axis of the cross section.

In order to satisfy the condition $F = 0$, we must make $\bar{y} = 0$. That is, the z axis of the cross section (labeled the z' axis in Figs. 6.10*a* and 6.11*a*) must pass through the centroid of the cross section. Thus, **the x axis passes through the centroid of each cross section of the undeformed beam.** The z' axis is called the *neutral axis of the cross section*, or simply the *neutral axis* (*NA*), because it is the boundary between the portion of the cross section that is in compression and the portion that is in tension, as indicated in Fig. 6.11.[4]



(*a*) Cross section. (*b*) Profile view.

FIGURE 6.11 (*a*) The location of the neutral axis of the cross section, and (*b*) the flexural stress distribution for a homogeneous beam in bending.

[4]In the future, the "z axis of the cross section" will just be labeled z, not z', even though the true z axis does not lie in the particular cross section under consideration.

Combining Eqs. 6.10b and 6.11c, we obtain the *moment-curvature equation* of Bernoulli-Euler beam theory, namely

$$M = \frac{EI}{\rho} = EI\kappa \qquad \textbf{Moment-curvature equation} \qquad (6.12)$$

The *curvature* $\kappa(x)$ is related to the *radius of curvature* $\rho(x)$ by $\kappa(x) = \dfrac{1}{\rho(x)}$. The product EI is called the *flexural rigidity* of the beam. (In Eq. 6.12 the subscript has been dropped from I_z to simplify the remainder of the discussion of bending of symmetric beams. Subscripts will be needed again in the Section 6.6 on Unsymmetric Bending.)

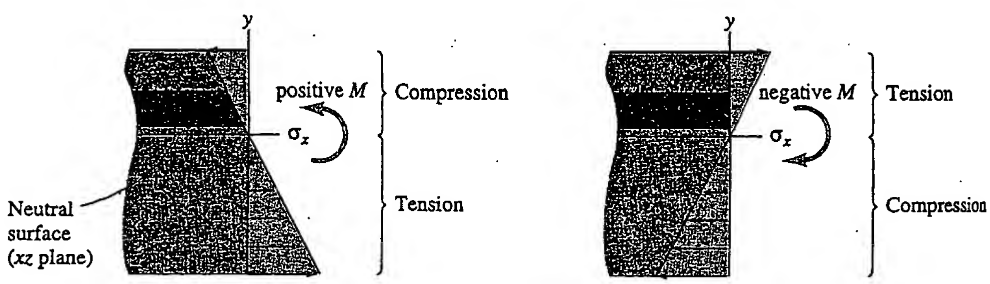
We can relate the moment-curvature equation, Eq. 6.12, to the deformed-beam segments in Fig. 6.8 by noting that a positive bending moment, $M(x)$, leads to a positive value of $\rho(x)$, which means that the beam is concave upward, as shown in Fig. 6.8*a*. Conversely, a negative moment produces a negative curvature, which means that the center of curvature lies in the $-y$ direction, as shown in Fig. 6.8*b*.

Finally, Eqs. 6.8 and 6.12 may be combined to give the important *flexure formula* of Bernoulli-Euler beam theory.[5]

$$\sigma = \frac{-My}{I} \qquad \textbf{Flexure formula} \qquad (6.13)$$

By making the assumptions that plane sections remain plane and that the material is linearly elastic with $E = E(x)$, we have obtained an expression for the stress distribution on a cross section subjected to bending moment $M(x)$. This is the linear stress distribution illustrated in Fig. 6.12.[6]

An assumption made in the derivation of the flexure formula, Eq. 6.13, is that σ_x is much greater than either σ_y or σ_z. It is left as an exercise for the reader to show that this is a reasonable assumption if the beam is long in comparison with its cross-sectional dimensions. (Homework Problem 6.3-36)



(*a*) Positive moment. (*b*) Negative moment.

FIGURE 6.12 The flexural stress distribution in a linearly elastic beam.

[5]According to the sign convention adopted in this text and illustrated in Fig. 5.4, a positive moment produces compression in the $+y$ fibers of the beam. This results in a minus sign in Eq. 6.13. Some textbooks adopt a different sign convention that leads to a plus sign in the flexure formula.

[6]Compressive stresses as well as tensile stresses may be shown acting on the cross section, as in Fig. 6.11*b*. However, to emphasize here that σ_x is linear in y, compressive stresses are shown in Fig. 6.12 as a continuation of the straight-line plot that depicts tensile stresses.